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# The Elastic Limits of Iron and Steel under Cyclical Variations of Stress

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# II. *The Elastic Limits of Iron and Steel under Cyclical Variations of Stress.*

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INTRODUCTION.—Since WÖHLER'S original experiments* on the fracture of iron and steel by repetition of stress, similar experiments have been made by independent observers, and all agree in showing that neither the maximum tensile strength nor the yield stress bears any simple relation to the range of stress which may be safely repeated.

The only theory of fatigue, *i.e.* of failure due to repetition of stress, which has received serious attention was put forward by BAUSCHINGER.† According to this theory, specimens subjected to repetitions of stress begin to be fatigued when the stresses applied in each cycle are so great that the extension of the specimen is not wholly elastic.

It is the object of the present paper to supply direct experimental information relating to suggestions made by BAUSCHINGER when applying his theory to WÖHLER'S results.

*Previous Work.*—BAUSCHINGER‡ repeated WÖHLER'S experiments in similar machines and made the additional observation of the stresses at which the specimens became inelastic. To determine the elastic limit in tension, the machine was stopped periodically and the specimen allowed to rest for a few hours, after which extensometer readings were taken with increasing loads until a deviation from HOOKE'S Law was found. The repetitions of stress were then continued, usually until fracture occurred.

As the present work is more comprehensive in its character than was BAUSCHINGER'S, it was found necessary to adopt a generalized expression for his theories. Cases occurred in which the usual terms "tension" and "compression" were unsuitable,

* W. C. UNWIN, "The Testing of Materials of Construction."

† J. BAUSCHINGER, 'Mittheilungen aus dem Mechanisch-technischen Laboratorium der k. Technischen Hochschule in München,' Heft XIII.

‡ J. BAUSCHINGER, 'Mittheilungen aus dem Mechanisch-technischen Laboratorium der k. Technischen Hochschule in München.' Gegründet von J. Bauschinger. Herausgegeben von August Föppl. Heft XXV.

and the two elastic limits are more conveniently referred to as "superior" and "inferior" respectively, compression being regarded as negative stress.

All the possible cases in which a material may be subjected to cyclical variations of stress are covered by the following theory:—

"The superior limit of elasticity can be raised or lowered by cyclical variations of stress, and at the same time the inferior limit of elasticity will be raised or lowered by a definite, but not necessarily the same, amount. The range of stress between the two elastic limits has therefore a value which depends only on the material and the stress at the inferior limit of elasticity. This elastic range of stress is the same in magnitude as the maximum range of stress which can be repeatedly applied to a bar without causing fracture, no matter how great the number of repetitions."

BAUSCHINGER found that these definitions did not apply to the elastic limits as measured on a previously unstrained specimen, and he made experiments to show that the elastic limits in this case, which he called the "primitive" elastic limits, were unstable, and that only a few reversals of stress were necessary to produce a condition in which the theory was satisfied.

In this latter state of the specimen BAUSCHINGER defined the elastic limits as "natural" elastic limits.

Other researches on fatigue to destruction have been described frequently.* Most of these consider only reversals of stress between approximately equal inferior and superior limits numerically, with an important exception in the original tests by WÖHLER. These tests and the machines used are fully described by Prof. UNWIN, and it will be found that for the three materials most carefully investigated by WÖHLER sufficient information is given for a curve to be obtained by plotting the minimum stress as abscissa and the corresponding safe ranges as ordinate. This was done by GERBER, who showed that a parabola agreed to a first approximation with the experimental results plotted in this way. If BAUSCHINGER's theory is correct, this parabola must also approximately represent the elastic ranges obtained under the same conditions. BAUSCHINGER's observations, however, so far disagreed with WÖHLER's that he finally rejected GERBER's parabola as untenable.

Microscopic examination, as furnished by the experiments of EWING and HUMPHREY,† supported BAUSCHINGER's theory in the case of equal and opposite stresses. On the other hand, preliminary experiments by Dr. STANTON and the author did not confirm BAUSCHINGER's observations.

* "On a Throw Testing Machine for Reversals of Direct Stress," O. REYNOLDS and J. H. SMITH 'Phil. Trans.,' A, vol. 199, p. 265; F. ROGERS, 'Journal Iron and Steel Institute,' vol. lxvii. (1905), p. 491; T. E. STANTON and L. BAIRSTOW, "On the Resistance of Iron and Steel to Reversals of Direct Stress," 'Min. Proc. Inst.C.E.,' vol. clxvi. (1903–6), part iv.

† "The Fracture of Metals under Repeated Alternations of Stress," J. A. EWING and J. C. W. HUMPHREY, 'Phil. Trans.,' A, vol. 200, p. 241.



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*The Present Experiments.*—Owing to the discrepancies just mentioned, further experiments appeared to be necessary in order to determine the laws of variation of the elastic limits of materials when subjected to cyclical variations of stress. An indication of the general results of the experiments will most clearly illustrate the course of the work.

It is found that, after a sufficient number of repetitions, iron or steel is capable of adjusting itself to variations of stress, cyclically applied. When this adjustment is complete, the specimen is found to have become perfectly elastic throughout the whole cycle, and fatigue does not occur.

This adjustment to a given cycle is possible because the limits of elasticity are not fixed, but can be raised or lowered by repetitions of stress.

During the adjustment of the elastic limits to a given cycle of stress a change of length occurs in the specimen which is the same as the extension observed in an ordinary tensile test when the yield stress is exceeded. For cyclically applied stress a similar extension occurs even when the maximum stress in the cycle is less than the static yield stress.

The greater the extension of the specimen during adjustment, the greater are the amounts by which the elastic limits are raised.

This power of adjustment is limited, and if the range of stress in the imposed cycle is sufficiently great, the specimen becomes or remains inelastic, and work is performed during each cycle. This work is expended in moving portions of the crystals relatively to one another, and is probably associated with microscopic slip-lines which gradually develop into cracks, ultimately causing the fracture of the specimen.

*Definition of Elasticity for Cyclical Variation of Stress.*—The definition of elasticity in a specimen subjected to repetitions of stresses is most conveniently expressed in reference to the cycle. If the elongation is proportional to the stress over the whole cycle, the specimen is elastic and the stress elongation diagram becomes a straight line.

When the specimen is inelastic, the straight line is replaced by a figure which closely resembles the familiar magnetic hysteresis loop. Such a loop EFGH is given in fig. 1, c, and will be in future referred to as a hysteresis loop.

*The Testing Machine and Extensometer.*—The testing machine was specially designed for the work, and was made in the Engineering Workshop of the National Physical Laboratory. For tensile loads the machine is of the usual type of single-lever testing machine for static tests. To obtain compressive stresses, a second system of knife-edges is used and the beam prolonged past the tension zero, so that when the jockey is moved back farther than the zero the specimen is put into compression.

Cyclical variations were produced by attaching two scale pans to the beam and automatically applying and removing the scale-pan weights. One of these weights regulated the maximum compressive stress, and the other the maximum tensile stress.

By combining the use of either of these scale-pan weights with that of the jockey, the machine was made suitable for tests in which the stress was never wholly removed.

The scale-pan weights were applied at regular intervals by means of hydraulic pressure operating through the specimen and raising and lowering each end of the

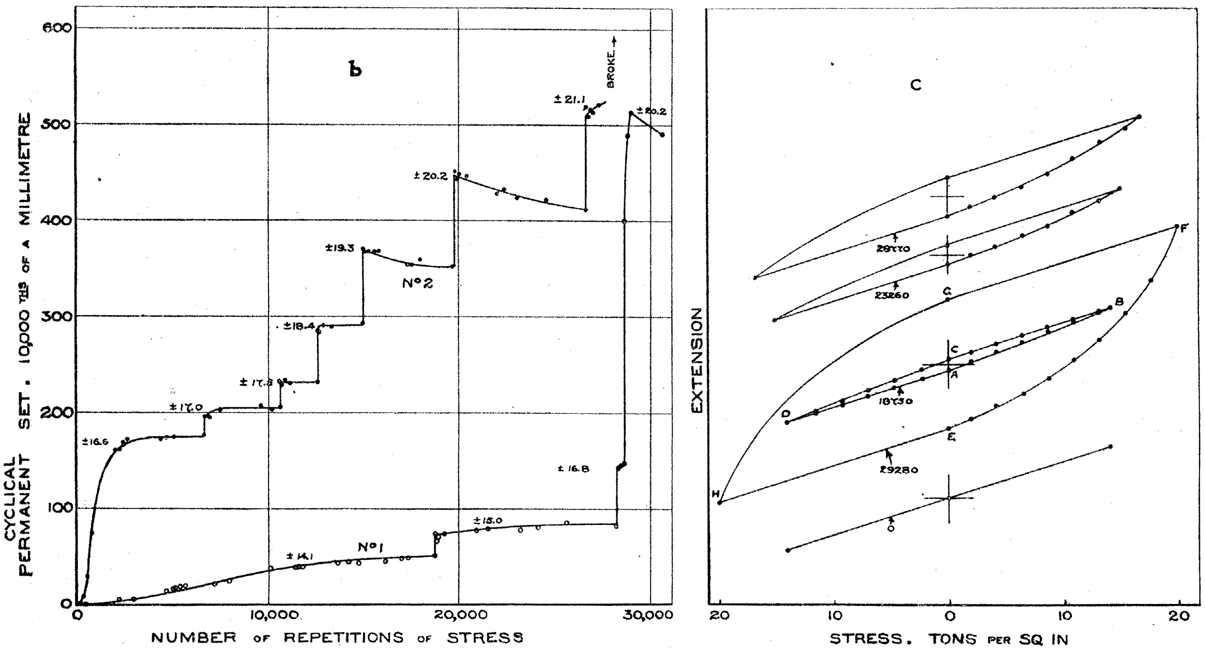


Fig. 1.

beam in turn. The valve controlling the water supply and exhaust was moved by link-gear driven from an electric motor, whilst the running of the motor itself was regulated by the movements of the beam and by a commutator attached to the link-gear.

The number of cycles was registered automatically.

Observations on the change of length of the specimen were made by means of the delicate extensometer designed by Prof. MARTENS. In this instrument, two knife-edge rhombs, each of which carries a mirror, are held against the back and front of the specimen by spring clips. The rotation of the mirrors is measured by observing the reflection of a fixed scale.

To render rapid simultaneous observations possible, an arrangement was adopted by means of which only a single telescope was necessary for making the readings corresponding to the movements of both mirrors.

*Materials used in the Research.*—Three samples of commercial iron and steel were obtained in the form of round bars, and complete analyses and tensile-test results are given in Tables I. and II.

The Swedish iron is that described by Dr. STANTON and the author in a paper on "The Resistance of Iron and Steel to Reversals of Stress."* A bar of Bessemer

* 'Min. Proc. Inst.C.E.,' vol. clxvi., 1905–6, Part iv.


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steel, similar in characteristics to No. 2 of the same paper, was taken as representative of hard material, whilst finally a bar of axle steel of intermediate resistance was obtained to complete the tests.

Both the harder materials were found to be exceedingly uniform in all respects, but the yield stress for the Swedish iron varied with the position in the bar from which the specimen had been cut. This variation had no apparent effect on the values of the elastic ranges, but the tests were discontinued when the change had become considerable.

The specimens, fig. 1A, were necessarily of small diameter and length. The portion

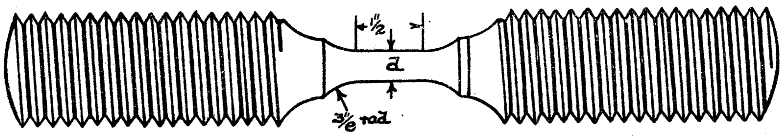


Fig. 1A.

under test was half an inch long in the parallel part, and the diameters, d, varied from 0·170 inch to 0·250 inch.

*The Observations.*—In the present experiments the measurement of the length of the specimen under the actual conditions of test has constituted an important new feature in the history of fatigue testing. Before the application of any stress the extensometer was fixed in position and a reading taken; this gave a zero from which all the changes of length were calculated.

As the changes which occur due to repetitions of stress are somewhat slow, observations were only taken at intervals of hours, when the cumulative effect of small variations had become large enough for measurement. The extreme points in any cycle could be observed without stopping the machine, and generally the readings at these extremes, *i.e.* at the maximum and minimum lengths of the specimen, were the only ones taken.

Occasionally the machine was stopped, and, starting from the minimum stress, the load was increased by successive increments to the maximum stress, the extension being read at each step. The load was then decreased and the corresponding extension measured in a similar manner. Plotting the extensions on a stress base then gave the straight line or loop which indicated the condition of the specimen as to elasticity.

It will be shown later, when the actual figures are described, that the maximum width of any loop, measured parallel to the axis of extension, added to the elastic extension of the specimen, is the total change of length in each cycle.

The complete plotting of the hysteresis loop was somewhat objectionable, as it meant an interruption of the usual running of the machine, and only a few complete loops were observed for each specimen. The intermediate readings are represented in the diagrams by the maximum width of the loop, defined as "cyclical permanent set," this width having been obtained by subtracting the calculated elastic extension from the total extension observed.

In addition to the cyclical changes of length just mentioned, non-cyclical extensions occurred, which took the form of a more or less rapid yielding of the material. It was in order to observe these non-cyclical extensions that the extensometer was kept in position during the whole test.* The change of length of the specimen which was non-cyclical has been defined as "permanent extension."

A detailed reference to a few actual readings will make the method of reduction clear. The new specimen was subjected to increasing loads, and for the first part of the test the extension was proportional to the load. From the modulus so obtained, the elastic extension for any range of stress could be calculated on the assumption that no change of modulus occurred. For a particular specimen of axle steel, the elastic extension was 344, the unit being the 10,000th part of a millimetre.

In the following table the actual scale readings for the same specimen are given at 10, 480, and 5330 repetitions :—

| Number of repetitions. | Maximum stress. | | Minimum stress. | |
|---|---|---|---|---|
| | Left-hand mirror. | Right-hand mirror. | Left-hand mirror. | Right-hand mirror. |
| 10 | 1240 | 1029 | 1068 | 1200 |
| 480 | 1240 | 1029 | 1068 | 1200 |
| 5330 | 1900 | 350 | 1710 | 538 |

At 10 reversals the scale reading for the left-hand mirror was 1068, when the minimum load was applied and gradually increased during the cycle until it became 1240 at the maximum load. The change of reading for this mirror was therefore 172. The change for the right-hand mirror was 171. The sum of these, *i.e.* 343, was the change of length during the cycle. This was almost exactly equal to the elastic extension of 344, and the effect of the reversals was not measurable.

All four readings were repeated after 480 repetitions, so that no change had been registered up to that time.

The readings at 5330 repetitions showed great changes in every case. The left- and right-hand mirrors now moved 190 and 188 divisions respectively, making a total change of length of 378. Of this, 344 is elastic and the remainder, 34, has been plotted in fig. 3 as the "cyclical permanent set" at 5330 repetitions.

The reading of 1900, obtained from the left-hand mirror at the maximum stress in the 5330th cycle, differed by 660 from the reading of 1240 obtained at the same

* The small amount of creeping of the extensometer which occurred appeared as a change of length. When suspected, the knife-edges of the extensometer were sharpened and a second specimen tried. The creep was generally unimportant.

stress during the 10th cycle. The readings of the right-hand mirror had changed 679 divisions. Adding the two changes together, it appeared that the specimen had increased in length by 1339 divisions. The "permanent extension" at 5330 repetitions is therefore 1339.

Obviously, all the readings might have been taken at the minimum stress, when the "permanent extension" would have appeared as 1304, or 35 divisions less. This difference is, of course, equal to the "cyclical permanent set" at the time. It does not appear to be possible to further separate the "cyclical permanent set" and the "permanent extension," but, fortunately, this is not important.

It has been usual in the experiments to keep the maximum stress constant and use the readings obtained at that stress for the calculation of "permanent extension." The only exceptions occurred in the cases when cyclical variations of stress between equal and opposite limits of stress were being observed. In these cases both loads were increased numerically by equal amounts, the mean stress being always zero. The mean of the extreme extensometer readings was then found to be practically constant.

It will be readily recognised that the method of approaching the limiting conditions by increasing the range from a small value is liable to be very costly and would only be justified if the supply of material were limited. It is usual, in making fatigue tests to destruction, to find any range which will break the specimen quickly, and then, decreasing the range for a new experiment, to gradually approach the limiting range. This may often be obtained from three or four cases of fracture by slight extrapolation. In a similar manner the observations in the present paper refer principally to cycles of stress greater than the limiting value for safety.

*The Results of the Experiments on Axle Steel.*—The observations are given graphically in figs. 1 to 5. Each figure corresponds to a different position of the elastic limits of the material, and the scales for corresponding curves have been maintained constant throughout the series, so that direct comparison can be made from one figure to another.

*Equal Tensile and Compressive Stresses* (fig. 1).—For equal stresses the observations were of the simplest kind. No permanent extension occurred and the hysteresis loop was quite closed. The changes in the specimen showed themselves entirely by the production of "cyclical permanent set," *i.e.* by an increase in the extension during each cycle.

After the specimen had been fixed in position, and before it had been loaded in either direction, a reading was taken of the unstrained length. A similar reading was recorded after the tension load had been applied and removed, and a third reading after putting on and removing the compression load. The three readings were alike and indicated complete elasticity, within the accuracy of measurement.

The stresses $\pm 14 \cdot 1$ tons per sq. inch were then repeated automatically, and for some time the straight line O in fig. 1, c, continued to represent the cycle of extensions. The cross indicates the origin of both extension and stress.

As the number of repetitions became greater, the "cyclical permanent set," curve No. 1, fig. 1, b, became measurable and gradually increased, until after nearly 19,000 reversals of stress it had become about 11 per cent. of the original elastic extension. The hysteresis loop marked 18,750 was then taken. Starting from A, a no-load point, the curve was traced to B as the tensile stress increased and fell to C as the tensile stress was removed. Exactly similar curves, CD and DA, apply to the compressive stresses. The parts BC and DA are parallel to the elastic line O, and it will be seen that there was no elastic limit in either tension or compression for increasing loads.

Raising the stresses to $\pm 15{\cdot}0$ tons per sq. inch produced an immediate increase in the "cyclical permanent set." The hysteresis loop at 23,260 reversals is similar to the one previously described.

Finally, stresses of $\pm 20{\cdot}2$ tons per sq. inch were imposed, and at 29,280 reversals diagram EFGH represented the condition of the specimen. The width of the hysteresis loop was then very great, but even for that case the lines FG and HE are parallel to the original elastic line O. It therefore follows, as has been previously stated, that the width of the hysteresis loop, *i.e.* EG, is equal to the change of length from H to F, minus the elastic extension, which can be calculated from the slope of the curve O and the range of stress applied.

The behaviour of this specimen illustrates the necessity for BAUSCHINGER'S hypothesis relating to "primitive" elastic limits, as the extensometer was incapable of showing the first deviations from elasticity. At a slightly lower range, probably $\pm 13$ tons per sq. inch, the specimen would have been really elastic, as no number of reversals would have produced a hysteresis loop. The general method of estimating this limiting range of elasticity will be fully given after the description of the results.

Some features of the Curve No. 1 are curious. At the lower ranges of stress the width of the hysteresis loop was scarcely affected by a considerable increase in the number of repetitions, whilst at the highest stress the specimen would appear to have supported reversals better after 1000 repetitions than at the earlier applications.

Another specimen (Curve No. 2, fig. 1, b) repeated the observations faithfully, and the type of result has been supported again and again. The figures near the curve give the stresses for each section. At stresses from $\pm 16{\cdot}6$ tons per sq. inch to $\pm 18{\cdot}4$ tons per sq. inch reversals during at least 24 hours produced no measurably increased effect, whilst at $\pm 19{\cdot}3$ and $\pm 20{\cdot}2$ tons per sq. inch recovery actually occurred. After only 1200 reversals at $\pm 21{\cdot}1$ tons per sq. inch the specimen broke. Half the fracture showed the absence of extension peculiar to alternating stress fractures.

Although the observations were continued almost to the actual breaking-point, the extensometer had given no special warning of the deterioration, and as extensions of about the 100,000th part of the length of the specimen could be detected, it will be realised how extremely local is the actual damage. Further, it would appear that individual slips in the crystalline grains cannot have increased their extent due to repetitions. The precise action which ultimately produced fracture is difficult to imagine.

*Unequal Stresses.*—The question here arose as to the best conditions of test. In the previous case of equal and opposite stresses the mean stress was kept constant, but it will be seen that this condition makes the experimental difficulties very great near the maximum tensile stress, where the greatest stress during variation cannot be increased appreciably. It was finally decided, after trial, to keep the maximum tensile stress constant for each experiment and to increase the range by reducing the minimum stress.

One immediate advantage of a constant maximum stress is that extensions which increase when the range is increased must be ascribed to the range and not to the maximum stress.

Keeping the maximum stress constant at +18·3 tons per sq. inch, a specimen (fig. 2) was subjected to repetitions of stress. The "cyclical permanent set," fig. 2, b,

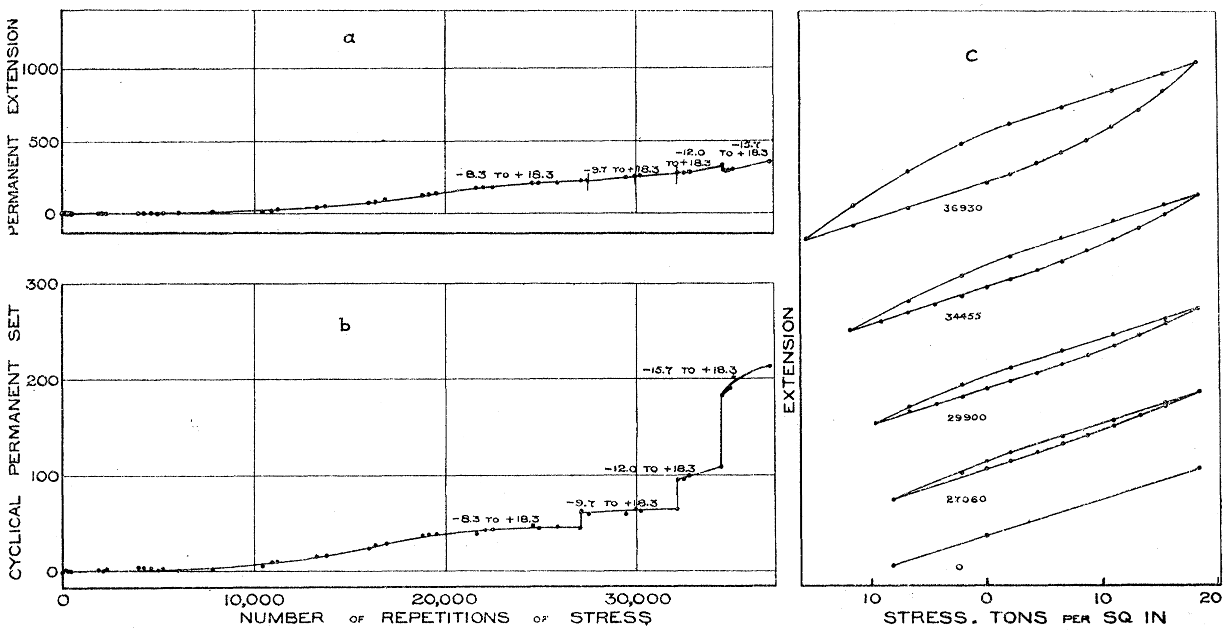


Fig. 2.

and the hysteresis loops, fig. 2, c, were similar to those previously obtained, the greatest width of the loop being at the mean stress.

The hysteresis loop was not, however, quite closed. The diagram, "permanent extension," fig. 2, a, shows the alteration of length from the original condition, all the extensometer readings having been taken when the stress was +18·3 tons per sq. inch.

The change in "cyclical permanent set" is thus seen to have been accompanied by a change of length, and when the "set" had become constant the rate of extension had also become constant and small, but not zero. The rate of extension during the day was almost always greater than that during the night, and this effect is supposed to be due to change of temperature as the magnitude of the change from day-rate to night-rate was greatest in frosty weather. The mean for the whole curve is regular, the earlier part of the curve having taken about 11 days for completion, involving a fairly good daily average.

Increase in the range of stress produced a greater rate of extension after a preliminary decrease due to the larger compression load. Equal stresses have been found, as previously stated, not to produce any appreciable permanent extension and, as the method tends to equality of loads, it would be expected that the rate of extension could not become very great in this case.

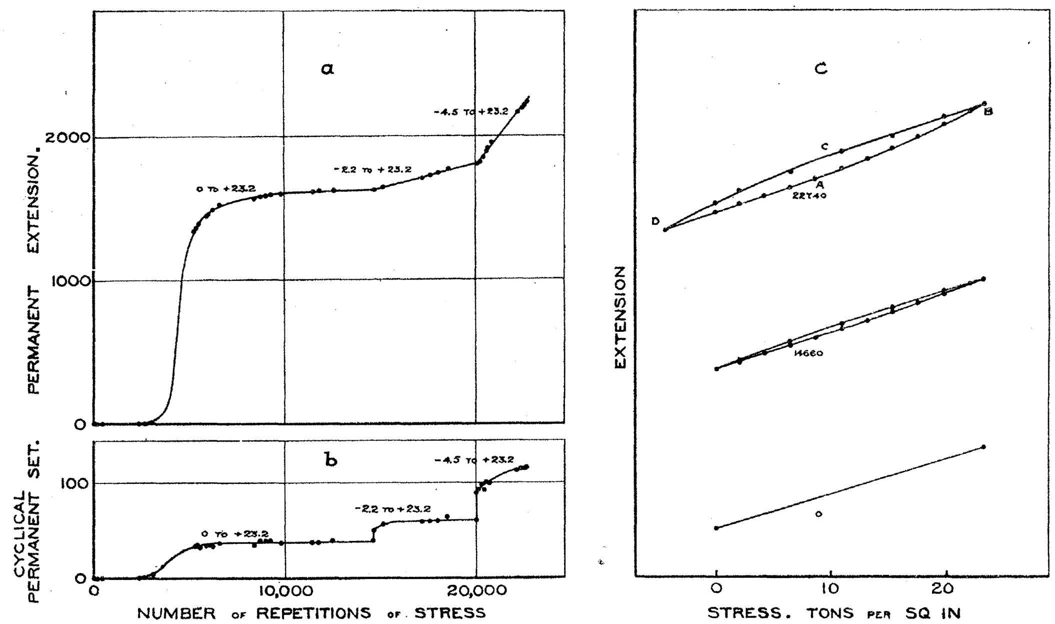


Fig. 3.

Fig. 3.—The maximum stress applied, *i.e.* +23·2 tons per sq. inch, was still below the ordinary yield-point for the material, and for the first 2000 repetitions very little appeared by the extensometer. Shortly afterwards somewhat rapid permanent extension began, fig. 3, a, which corresponded to a slow yield, and at the same time a hysteresis loop, fig. 3, c, made its appearance. The latter reached its maximum width in about 7000 repetitions and then remained unaltered for a further 8000 repetitions. During this latter time the rate of extension was gradually becoming less, until finally it was very small. A slightly decreased range would have allowed this part of the curve to become horizontal and the conditions would have been stable. Some "cyclical permanent set" might then have remained temporarily, but there is every reason to believe that it would have disappeared by recovery if sufficient time were allowed.

In the case of complete recovery an extension of about 0·2 mm. would have been produced in the process of raising the superior elastic limit from 13 tons per sq. inch to 23 tons per sq. inch. In the actual case the range is just too great and the specimen is incapable of completely adjusting itself to an elastic state.

An increase in the range, by making the stresses −2·2 tons per sq. inch and +23·2 tons per sq. inch, immediately put up the "cyclical permanent set" and increased the rate of extension. A further increase in the range produced further increases in both quantities.

The hysteresis loop and the "cyclical permanent set" again agreed with those of figs. 1 and 2, the loop being symmetrical about the mean load.

The numbers plotted under "cyclical permanent set," fig. 3, b, are now greater than the width of the hysteresis loop. Obviously, the increased length of the specimen, and consequent decreased diameter, will affect the elastic extension so as to make it greater than that calculated from the original specimen. When correction is made for this, the lines BC and DA become parallel to the elastic line, within the probable errors of observation, and, although scarcely any compressive stress occurs, the hysteresis loop could not easily be distinguished from a similar loop produced by equal stresses.

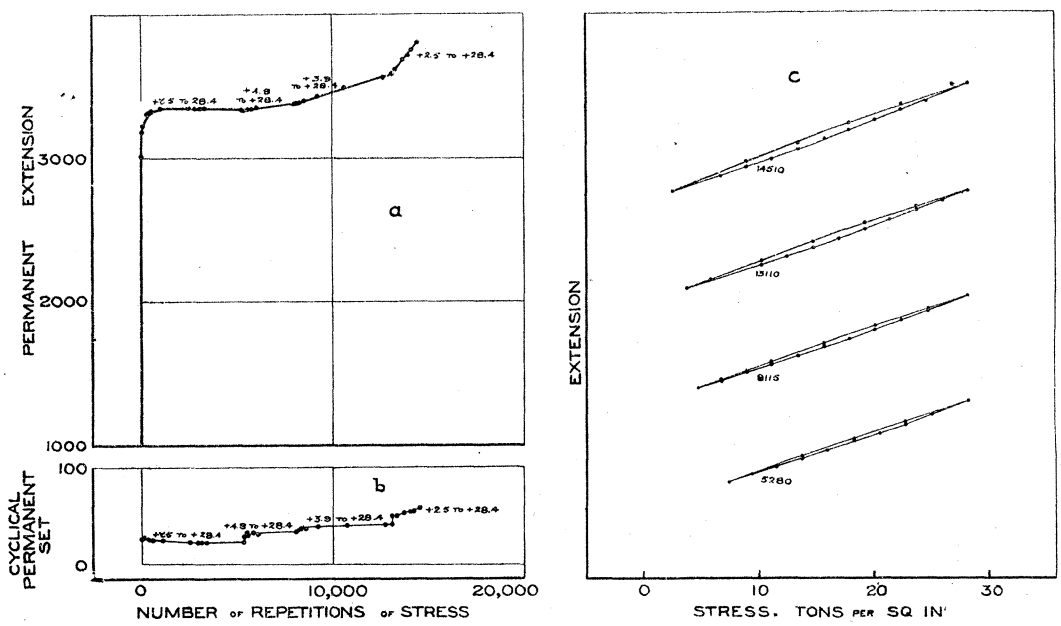


Fig. 4.

Fig. 4.—Except that the maximum stress is now 28·4 tons per sq. inch, and that the yield occurred with the first load, the description of the last figure applies very closely to the general features. The first range of stress applied was insufficient to produce progressive extension, and the "permanent extension" curve even receded slightly. It will be noticed, also, that some recovery occurs in the "cyclical permanent set." About half the remainder of this quantity is accounted for by the increased elastic



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extension due to the yield. The actual hysteresis loop was very narrow and was not symmetrical about the mean load. Probably, after the lapse of a long time, the specimen would have become elastic for this range. The "permanent extension" is much greater than that described in fig. 3, and the corresponding change in the position of the superior elastic limit is also greater, amounting to 15 tons per sq. inch.

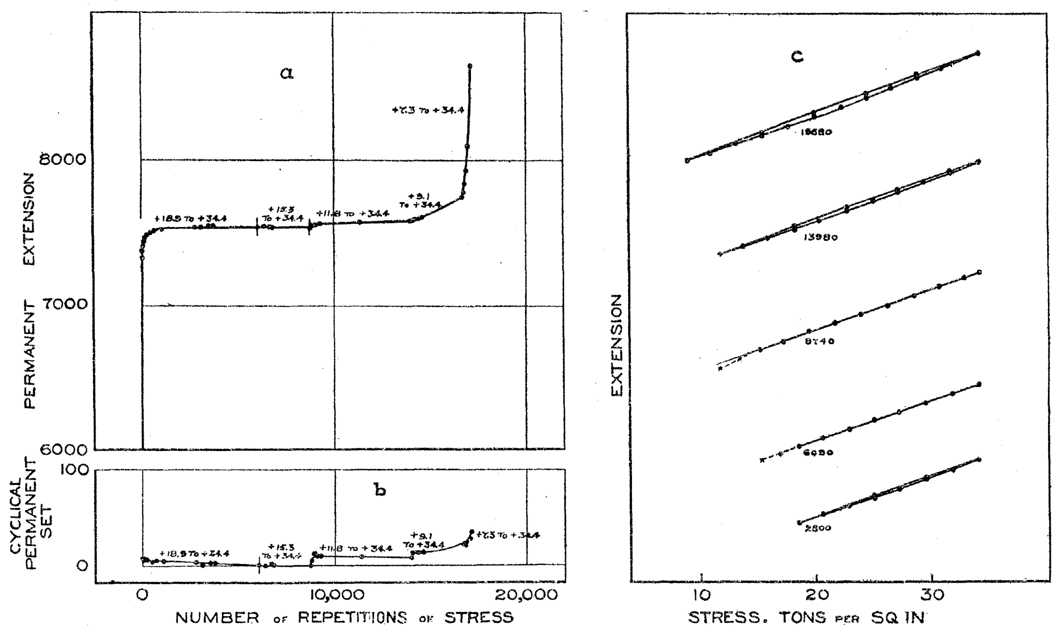


Fig. 5.

Fig. 5.—The maximum stress to which the specimen was subjected was in the neighbourhood of the maximum tensile stress, and it will be seen that very great differences in the minimum stress were necessary to produce measurable effects. The first large extension was accompanied by a hysteresis loop, fig. 5, c, which gradually decreased in width until at 6000 repetitions the differences from elasticity were not greater than the errors of observation. A new elastic line was therefore found which corresponded with the extended length, and this has been taken as the standard for the calculation of the modulus of elasticity for the higher loads.

The slope of this line again agreed with the corrected value obtained from the new specimen.

The "permanent extension," fig. 5, a, ceased at 6000 reversals and was not affected by an increase of range of 3·6 tons per sq. inch, nor was any hysteresis loop produced, fig. 5, c. Further increase of range immediately produced a "cyclical permanent set," fig. 5, b, and slow extension commenced. Finally, with a range of stress of 27·0 tons per sq. inch, very rapid extension set in. Even for this case, however, the hysteresis loop was not very great.

The superior elastic limit has now been raised by extension, almost up to the maximum tensile stress, and further extension would raise the inferior limit of elasticity very rapidly, until finally it would reach the maximum tensile stress of the material when the superior limit reached the same point; the range being then zero.

*General Remarks on the Diagrams.*—When a specimen was being fatigued by the application of an unsafe range of stress, the hysteresis loop produced retained the same general shape for all ratios of the maximum and minimum stresses.

The two parts of the loop which were traced out, as the stress varied from either extreme of the cycle towards the mean stress, were straight lines with a slope equal to that which would have been given by the specimen if restored to an elastic condition. Further than this, no appreciable change occurred in the value of YOUNG'S modulus.

The increase in width of the hysteresis loop, for a given increase in the range applied, was greatest for the case of equal and opposite stresses, and gradually decreased in such a way as to suggest that at the maximum stress there was no tendency to form such a loop. The ordinary tensile test only differs from this extreme case in the fact that a short time only is allowed for fracture, whilst an indefinitely long one would be required as an extension of the case of varying stress. This difference is unimportant, as careful experiments have shown that the rate of test has little effect on the maximum stress obtained.

The rate of permanent extension due to excess range, after the adjustment of the elastic limits, is not so clearly shown. It is zero for the case of equal stresses and becomes very great in the case of failure under high maximum stresses.

*The Determination of the Elastic Ranges for the Materials from the Observations.*—Before the numerical values of these are given one further point of interest must be noticed. Some specimens of Swedish iron and axle steel showed a large "permanent extension" which ultimately ceased. In all cases "cyclical permanent set" was produced by the initial "permanent extension," the amount of which decreased as the test proceeded and in some cases disappeared. It would therefore seem that an extension produced by repetitions of stress introduced a temporary want of elasticity during each cycle, and that a specimen which is, in reality, quite safe may appear inelastic.

In further support of this statement may be mentioned a specimen which has been described elsewhere.* A sample of Swedish iron was subjected to reversals of stress of $-8 \cdot 3$ tons per sq. inch and $+8 \cdot 9$ tons per sq. inch at 1200 r.p.m. Due to some cause, possibly the inequality of the stresses, a hysteresis loop was produced after 100,000 reversals, and this remained even after a million reversals, the specimen being then unbroken. After many months of rest the elasticity was completely restored over the whole range. One million reversals at the slightly higher range of 18·2 tons per sq. inch would have produced fracture.

It may possibly be that the decreased range of stress found by REYNOLDS and SMITH has some relation to the question of recovery, but further experiments are

* 'Report of the National Physical Laboratory,' 1907, p. 58.

necessary to decide the question, as the effect of the rigidity of the testing machine has not yet been fully investigated. In two instances, at least, low ranges of stress have been traced to natural periods of vibration in the testing machine agreeing approximately with the period of repetition.

The fact that the hysteresis loop is not necessarily a sign of failure increased the difficulty of giving numerical values to the elastic ranges.

If the widths of the hysteresis loops are plotted as ordinates, on a range of stress base, it will be seen that the curve obtained is approximately a straight line, particularly for the lower ranges of stress. This line produced backwards intersects the axis of range of stress. As the ordinate is zero at the intersection, the hysteresis loop must disappear, or, in other words, the specimen must be completely elastic.

The actual prediction was complicated by the fact that the hysteresis loop did not always attain a constant width for the range. In such cases the width of loop was taken immediately after the preliminary change in "cyclical permanent set," shown most clearly in the upper curve of fig. 1.

In most cases the "rate of permanent extension" afforded a check on the result, as the prediction could be made by substituting this "rate" for the "cyclical permanent set" and using the same method of extrapolation.

Even with the greatest care the uncertainty of the correction to be applied was too great to allow of much extrapolation, and the earliest results were always obtained as close to the limiting conditions as possible.

The ranges of elasticity are given to the nearest $\frac{1}{2}$ ton per sq. inch, which is about the limit of accuracy expected.

*The Yielding of Iron and Steel.*—The earlier portion of the permanent extension is of considerable interest. Except for the case of complete reversals of stress, the application of a range of stress slightly in excess of the safe range produced an elongation of the specimen, whether the maximum stress was above the yield-point or not. This is shown in fig. 6, where the information necessary has been collected and expressed graphically.

The ordinates are the maximum stresses applied to the specimen and the abscissæ the corresponding permanent extensions during the adjustment of the superior elastic limits to the maximum stresses. Starting with a new specimen, the line FEA shows that, at a stress of 25 tons per sq. inch, no permanent extension was observed. When the load was slightly increased, a sudden extension of about one-fifth of a millimetre occurred, this being the well-known yield. Further increase of stress extended the specimen still further, the changes being represented by a line which cannot differ appreciably from BC. In producing the curve FEABC no cyclical variations of stress are concerned, and the curve is identical with the usual stress elongation diagram frequently taken during a tensile test.

The experiment on the specimen of axle steel described in fig. 3 showed that under cyclical variations of stress an extension, which was not measurable at the first application of the load, gradually appeared due to repeated applications of a range of stress slightly greater than the safe range. This extension continued for some time, the dotted line GH representing the slow yielding of the specimen. When the adjustment of the elastic limits was complete, H represented the final extension, and no further increase of length occurred due to further repetition of stress.

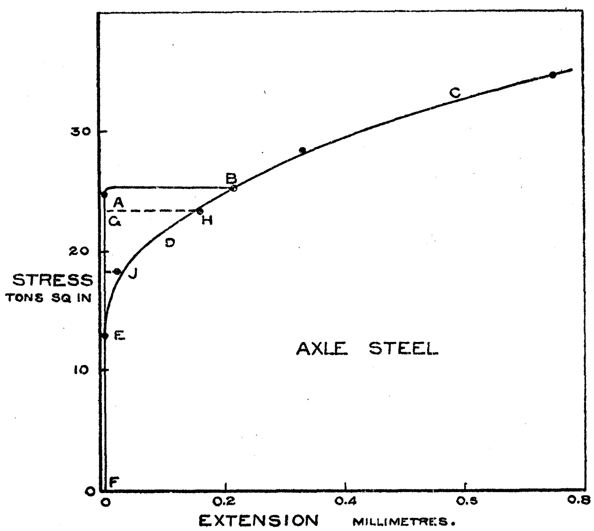


Fig. 6.

The point J was similarly obtained by repeating a cycle of stress having a less maximum value than that which produced the extension H. At E, which corresponds to the maximum safe stress during reversals, no extension occurred.

The points H, J, and E are evidently on a continuation of the curve BC, and when cyclical variations of stress are considered there is no break, in the curve at B, corresponding to the static yield-point.

Above this latter point the whole extension is produced by the maximum stress only, independently of the range of stress, which may be zero. As HJE is continuous with BC, it seems possible that an extension such as GH may be produced by the repetition of a cycle of stress in which the range is less than the safe range.

Should further experiments bear out this contention, it seems that very great care must be exercised in the use of materials having a higher maximum stress than that corresponding to alternations of equal and opposite stresses.

Below the static yield-point, iron and steel appear to be capable of maintaining an unstable condition for a considerable time against cyclical variations of stress which ultimately produce a considerable change of length. The first application of the maximum stress in a given cycle may show only a scarcely measurable extension, in spite of the fact that extension of thousands of times the amount may be obtained without any change in the cycle of stresses. It would appear from this that BAUSCHINGER's definitions of the "primitive" and "natural" elastic limits are in reality statements that ordinary extensometers are not sufficiently sensitive to detect the first signs of want of elasticity, and that fatigue increases these signs to recognisable magnitude.

*The Swedish Charcoal Iron and Bessemer Steel.*—These materials were dealt with first, and the observations are in some respects less complete, as clear ideas had not then been reached. The difficulties of experiment become greater as the steel gets harder, the duration of the test being longer and the rate of recovery due to time less rapid.

For the Swedish iron the results were obtained with a fresh specimen for each range of stress. The hysteresis loop did not become so constant in width, but otherwise the experiments are in complete accord with those for the axle steel. One of the specimens showed the whole of the extension up to fracture. It seems, therefore, unlikely that anything of importance has been missed in the history of a fatigue test. The Marten's extensometer is too delicate an instrument for use when fracture is anticipated, and a less accurate extensometer was substituted in such cases.

One series of experiments was made with the maximum stress very near to the breaking stress, and again a case of actual fracture, observed nearly to the end, showed the phenomena which may be expected to occur, and gives an indication as to when observations have proceeded far enough.

Recovery due to stoppage of the machine or other causes was somewhat rapid for this material.

The results for the Bessemer steel are very similar to the axle steel in all respects, the ranges chosen being in closely corresponding positions. Some slight differences might, perhaps, be referred to. In the case of equal stresses, the initial breakdown of the unstable condition was not succeeded by a nearly horizontal portion; in other words, the width of the hysteresis loop continually increased. This is not an important distinction, but is more easy to understand than is the behaviour of the axle steel under the same conditions.

The earliest case of unequal stresses was treated somewhat differently from the axle steel. The first range of stress applied produced a gradual extension of the specimen, and, after the rate of extension had reached its maximum, recovery was so very slow that finality was not nearly reached after 17,000 repetitions. The range was then decreased. From this point the observations agree with those on axle steel. The rate of extension for the initial range, when this was returned to, was considerably less than that previously observed. The question of recovery is obviously an important factor in the behaviour of materials which are being fatigued.

The remainder of the experiments do not call for special mention as no points of difference from the axle steel occurred.

*The Elastic Ranges of the Materials.*—The whole of the results are given in fig. 7, together with similar figures for WÖHLER's experiments on fatigue to destruction.

For two of these materials tests to fracture were made at moderately high speeds.

At 800 r.p.m. the Swedish iron gave a safe range of 19·0 tons per sq. inch and at 1200 r.p.m. of 18·2 tons per sq. inch. The elastic range now found at 2 r.p.m. is 19 tons per sq. inch, which is in close agreement.

The Bessemer steel was not tested at 800 r.p.m., but gave a range of 26·5 tons per sq. inch at 1200 r.p.m. A similar material at 800 r.p.m. gave a range of 30·5 tons per sq. inch. The elastic range now found at 2 r.p.m. is 33 tons per sq. inch. Here the effect of speed would appear to agree with that found by REYNOLDS and SMITH, the differences being too great for errors of observation.

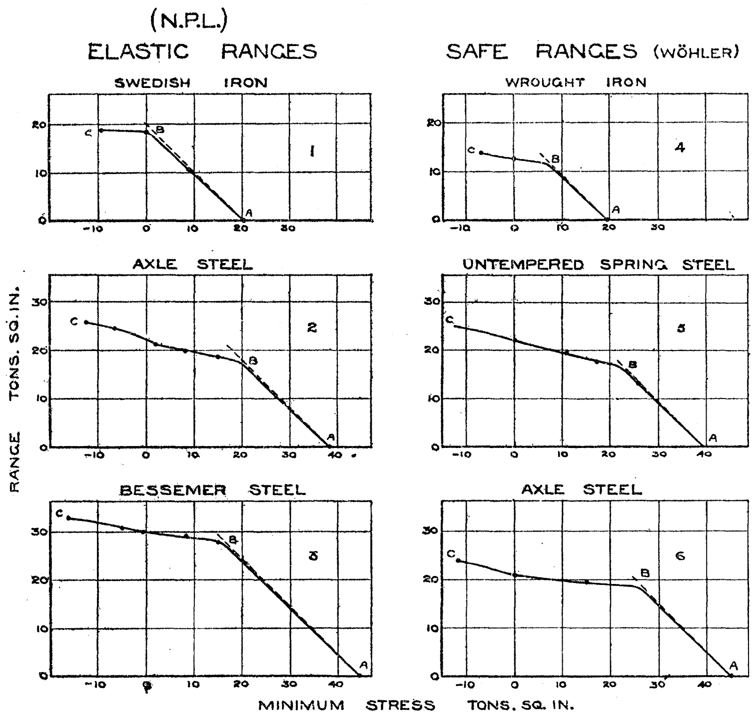


Fig. 7.

The abscissa for each curve of fig. 7 is the stress at the inferior limit of elasticity, compressive stresses being counted as negative. The ordinate is the elastic range of stress, corresponding to the minimum stress represented by the abscissa.

The value of the abscissa for the point where the ordinate is zero is the ordinary tensile maximum stress, as has been pointed out by GERBER. A line inclined at 45° to both axes and passing through this point forms an upper limit to the possible curve obtainable, for at any point on this line the superior limit of stress is equal to the tensile maximum stress. This line, AB, is shown dotted in all the curves.

For the reason given above the portion of the curve CB cannot cross the line AB, and, after the points have been carried close to the latter limit, the whole series of observations is completed by taking the maximum stress as the upper limit of elasticity. This, of course, is not strictly true, but the degree of approximation is clearly indicated by both the Swedish iron and Bessemer steel. The slope of the line CB is greatest for the axle steel, but otherwise the general features of the curves for all three materials are the same.

The remaining three curves have been drawn through Wöhler's points of observation. The wrought iron differs from the Swedish iron only in the magnitude of the ordinate. Curve 5 is almost identically the same as Curve 2 for axle steel, but Wöhler's points are not sufficiently numerous to complete the curve with certainty, and for these observations Gerber's parabola can be made to fit with equal accuracy.

The last of the curves again completely represents Wöhler's results, and considering the three diagrams together the agreement is remarkably close.

The truth of Bauschinger's theories would now appear to be established, as the elastic ranges have been shown to agree with Wöhler's safe ranges. The experimental results, however, show very different features to those obtained by Bauschinger, and this is probably due to the more complete observation of the extensions and, in part, to the different definition of elasticity adopted.

*The Condition of Specimens after Recovery from Fatigue.*—A suggestion made by Bauschinger as to a rapid means of determining the "natural" elastic limits seemed worth investigation. He suggested that a specimen be made inelastic by overloading, and then by increasing the range from a small value the maximum range of elasticity be found as recovery proceeded. The only experiments given involved less than one dozen reversals and little time was allowed for recovery. It is not surprising, therefore, that under these conditions the elastic ranges had no resemblance to the safe ranges found later.

Muir has since shown that very complete recovery of elasticity can be produced by the immersion of a specimen in boiling water for a few minutes. Taking advantage of this, a series of tests was made on some of the specimens already described. Recovery reduces the permanent set at a given load, as will be seen from one of the Bessemer steel specimens which was given a rest of 45 hours at the end of an experiment; almost exactly one-third of the set disappeared in the interval. A specimen after fatigue therefore tends to return to its primitive condition, and it is necessary to have experimental evidence to show the extent of the recovery.

The detailed observations are given in Table III. At the stresses indicated by the first column, extensometer readings were taken and the extensions are tabulated in the succeeding columns. The main heading relates to the recovery produced immediately before the experiment, whilst the letters T and C signify tension and compression respectively. The elastic limit for each experiment is indicated by heavy-type figures in the column of extensions. The general features are most easily seen from the plotted curves, as the errors of observation are then more easily allowed for. These are never great in absolute magnitude, but they may make individual differences appear unduly important.

*Axle Steel.*—The original condition of this material showed no want of elasticity in tension until within a short distance of the yield. Specimen No. 2, fig. 1, showed, also, that in compression the elastic limit was higher than 17 tons per sq. inch. The apparent elastic range was therefore greater than 42 tons per sq. inch as against the 26 tons per sq. inch which was maintained against repetitions.

Experiments Nos. 2, 3, 4, 5 and 6, Table III., refer to a specimen in the condition of Specimen No. 2 after 6000 reversals.

Following a final load of 18·26 tons per sq. inch in tension, the specimen was heated in boiling water for 15 minutes. Test No. 2 showed that the specimen was still elastic at the same load. In compression the limit was reached at about 8·5 tons per sq. inch and the load was continued to 13·28 tons per sq. inch. Recovery was again produced, and it was then found that the elastic limit in compression had been raised at least to 13·28 tons per sq. inch, but that the tensile elastic limit had fallen to 13·0 tons per sq. inch. By alternately heating and testing, the elastic limits were moved about very considerably, but always with the condition that if the tension limit was raised the compression limit was depressed, and *vice versâ*.

Both limits cannot be determined at the same time, but Test 6 shows that the elastic limit was found at the last stress imposed before heating, and the elastic ranges for the four previous cases can now be seen to be as nearly as possible 26 tons per sq. inch, *i.e.* equal to the elastic range found by repetitions. The agreement is not complete, as no extension occurs to correspond with the "permanent extension" of figs. 2 and 3.

Specimen 5 was immersed in boiling water for an hour and a half, without any other treatment between this heating and the repetitions of stress recorded in fig. 3. The elastic limit in tension now appeared at 20 tons per sq. inch, the loading being carried to 26·6 tons per sq. inch. On heating and retesting the limit was again found at 20 tons per sq. inch, and so on for Tests 9 and 10. The compression limit found in Test 11 is zero, but is uncertain, due to the smallness of the readings. The elastic limit of 20 tons per sq. inch, approximately, is somewhat less than the 22 tons per sq. inch estimated from the repetition experiments.

The two specimens just described illustrate BAUSCHINGER'S two theorems very clearly, the first showing that for nearly equal stresses the elastic limit in tension can only be raised by a corresponding drop in the compression limit, whilst the second shows that a limit exists above which the tension limit cannot be raised, so long as the stress is entirely removed in each cycle.

Specimen No. 7, fig. 5, of the axle steel should agree with Specimen No. 5, as the last load applied was zero. Test 12 did not show any limit until 26·6 tons per sq. inch had been reached, and then the departure from the elastic line was small. The compression limit appeared to be 5 tons per sq. inch, or the elastic range was apparently 31 or 32 tons per sq. inch. As the method of repetition gave 19 tons per sq. inch range, the specimen was no better than in its primitive condition.

The experiments for Swedish iron, made with nearly equal stresses, agreed with the axle steel under the same conditions, the elastic range determined by test and recovery agreeing very closely with that found by repetitions of stress. On the other hand, the Bessemer steel showed an elastic range 33 per cent. greater than that which could be maintained against repetitions. Possibly a more sensitive extensometer, or a longer specimen, would have yielded results in accordance with the earlier materials, but no means of testing this were available. Until further experiments have been made, this method of finding the elastic ranges by producing fatigue and recovery is uncertain.

Specimen No. 7 of the axle steel seems to indicate the possibility that the great deformations produced during rolling cause the high primitive elastic limits which are always found in bars of iron and steel.

In conclusion the author wishes to express his great indebtedness to Dr. STANTON for his interest and help, and for the opportunity of carrying out a research which he had proposed and commenced. He also begs to thank Dr. GLAZEBROOK, F.R.S., for the interest which he has shown in the work, and for the means of carrying on the experiments.

TABLE I.—Tensile Tests of the Materials Used.

| | Yield-point, tons per sq. inch. | Maximum stress, tons per sq. inch. | Elongation on 2 inches. |
|---|---|---|---|
| | | | per cent. |
| Swedish charcoal iron . . . | 14·5 | 19·6 | 54·0 |
| Axle steel . . . . . . . . | 24·9 | 38·2 | 33·5 |
| Bessemer steel . . . . . . | 32·0 | 44·6 | 20·6 |

TABLE II.—Analyses of the Materials Used.

| | Carbon. | Manganese. | Silicon. | Sulphur. | Phosphorus. | Arsenic. |
|---|---|---|---|---|---|---|
| | per cent. | per cent. | per cent. | per cent. | per cent. | per cent. |
| Swedish iron . . . | 0·039 | Trace | Trace | 0·006 | 0·018 | — |
| Axle steel . . . . | 0·347 | 0·747 | 0·093 | 0·039 | 0·051 | 0·013 |
| Bessemer steel . . . | 0·460 | 0·275 | 0·037 | 0·009 | 0·017 | 0·014 |

TABLE III.—Axle Steel.

| Exp.… | 1. | 2. | | 3. | | 4. | | 5. | | 6. | 7. | 8. | 9. | 10. | 11. | | 12. | 13. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | Specimen No. 5. | | | | | | Specimen No. 7. | | |
| Stress, tons per sq. inch. | Original condition. | Heated for 15 minutes at 100° C. | | Heated for 15 minutes at 100° C. | | Heated for 15 minutes at 100° C. | | Heated for 15 minutes at 100° C. | | Heated for 15 minutes at 100° C. | Heated for 1·5 hours at 100° C. | Heated for 15 minutes at 100° C. | Heated for 15 minutes at 100° C. | Heated for 15 minutes at 100° C. | Heated for 15 minutes at 100° C. | | Heated for 1·5 hours at 100° C. | Heated for 20 minutes at 100° C. | |
| | T. | T. | C. | C. | T. | T. | C. | C. | T. | T. | T. | T. | T. | T. | T. | C. | T. | T. | C. |
| 1·66 | | | 24 | | 26 | | 23 | | 24 | 24 | | | | | | 29 | 24 | | 27 |
| 3·32 | | 48 | 49 | 51 | 49 | | 47 | 50 | 49 | 47 | 49 | 51 | 50 | 50 | 50 | 53 | 52 | 53 | 56 |
| 4·98 | 69 | | 75 | | 74 | 72 | 73 | | 73 | 70 | | | | | | 78 | 80 | | **85** |
| 6·64 | | 97 | 100 | 98 | 98 | | 100 | 97 | 99 | 94 | 99 | 95 | 100 | 97 | 99 | 109 | 105 | 103 | 111 |
| 8·30 | | | **126** | | 122 | 120 | **128** | | 124 | 121 | | | | | | 149 | 130 | | 146 |
| 9·96 | 143 | 146 | 156 | 149 | 148 | | 151 | 148 | **150** | 146 | 148 | 148 | 146 | 149 | 149 | 205 | 157 | 157 | 186 |
| 11·62 | | | 184 | | 175 | 168 | 184 | | 183 | 167 | | | | | | | 185 | | 241 |
| 13·28 | | 194 | 222 | **198** | **198** | | 213 | 194 | 211 | 189 | 196 | 195 | 199 | 197 | 197 | | 210 | 211 | 323 |
| 14·94 | 216 | | | | 226 | 216 | 251 | | 249 | **217** | | | | | | | 237 | | |
| 16·60 | | | | | 258 | | 307 | **244** | | 245 | 248 | 248 | 247 | 244 | 250 | | 262 | 263 | |
| 18·26 | | **271** | | | 307 | **266** | | | | 283 | | | | | | | 291 | | |
| 19·92 | 289 | | | | | | | | | 347 | **294** | **296** | **300** | **298** | **299** | | 316 | 317 | |
| 21·58 | | | | | | | | | | | | 329 | 330 | 325 | | | 339 | | |
| 23·24 | | | | | | | | | | | 350 | 356 | 360 | 357 | | | 370 | | |
| 24·90 | **362** | | | | | | | | | | | 394 | 397 | 388 | | | 396 | | |
| 26·56 | | | | | | | | | | | 448 | 430 | 432 | 422 | | | **422** | | |
| 28·22 | | | | | | | | | | | | | | | | | 451 | | |
| 29·88 | Yielded | | | | | | | | | | | | | | | | 479 | | |
| 31·54 | | | | | | | | | | | | | | | | | 511 | | |
| 33·20 | | | | | | | | | | | | | | | | | 542 | | |
| 34·88 | | | | | | | | | | | | | | | | | 579 | | |